

Exercise 15

Determine if the systems in Exercises 15 and 16 are consistent. Do not completely solve the systems.

$$\begin{aligned}x_1 + 3x_3 &= 2 \\x_2 - 3x_4 &= 3 \\-2x_2 + 3x_3 + 2x_4 &= 1 \\3x_1 + 7x_4 &= -5\end{aligned}$$

Solution

Write the augmented matrix corresponding to this system of equations.

$$\left[\begin{array}{cccc|c} 1 & 0 & 3 & 0 & 2 \\ 0 & 1 & 0 & -3 & 3 \\ 0 & -2 & 3 & 2 & 1 \\ 3 & 0 & 0 & 7 & -5 \end{array} \right]$$

The aim is to write this in triangular form. To make the bottom left entry 0, multiply the first row by -3 and add it to the fourth row.

$$\left[\begin{array}{cccc|c} 1 & 0 & 3 & 0 & 2 \\ 0 & 1 & 0 & -3 & 3 \\ 0 & -2 & 3 & 2 & 1 \\ 0 & 0 & -9 & 7 & -11 \end{array} \right]$$

Multiply the second row by 2 and add it to the third row.

$$\left[\begin{array}{cccc|c} 1 & 0 & 3 & 0 & 2 \\ 0 & 1 & 0 & -3 & 3 \\ 0 & 0 & 3 & -4 & 7 \\ 0 & 0 & -9 & 7 & -11 \end{array} \right]$$

Multiply the third row by 3 and add it to the fourth row.

$$\left[\begin{array}{cccc|c} 1 & 0 & 3 & 0 & 2 \\ 0 & 1 & 0 & -3 & 3 \\ 0 & 0 & 3 & -4 & 7 \\ 0 & 0 & 0 & -5 & 10 \end{array} \right]$$

Since a solution for x_1 , x_2 , x_3 , and x_4 exists, the system is consistent.